

How to measure the speed of gravity

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August 14, 2012

Expanded version of essay written for and which received honourable mention in the Gravity Research Foundation 2012 Awards for Essays on Gravitation

Abstract

We propose a simple laboratory experiment to measure the speed of propagation of gravitational phenomena. We consider two masses placed at two different distances from a detector of gravitational force. They are placed so that the static gravitational force they produce at the detector exactly cancels. We show that if the masses are made to oscillate, then the force on the detector no longer vanishes and is proportional to the relative time delay it takes for the propagation of the gravitational changes from the masses to the detector. This time delay is inversely proportional to the speed of gravity. A measurement of the finiteness of this speed would be the first confirmation of one dynamical aspect of Einstein's theory of general relativity, that gravitational effects are not instantaneous and propagate at a finite velocity and a direct confirmation of the fundamental notion that there is no action at a distance. It is fully expected that the speed of gravity is equal to the speed of light.

KEY WORDS: Gravity, propagation speed

1 Introduction

Static gravitational phenomena has been described by the theory of Newton since the 17th century with striking accord with observations in the

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solar system and in the laboratory. In the early 20th century this theory was superseded by the dynamical theory of Einstein, which is usually called general relativity. General relativity is considered one of the summits of human intellectual advances and revolutionizes our understanding of the nature of gravitation and its interaction with matter, and of the nature of space and time. General relativity was successful in explaining certain kinematical discrepancies between Newton's theory and observations, especially the bending of starlight by gravitating bodies and the surfeit in the advance of the perihelion of Mercury. Up to now, direct verification of the theory of general relativity has been restricted to its kinematical predictions, that matter causes the curvature of space-time and material bodies move along geodesics of the curved space-time manifold. However, general relativity also predicts the complete dynamical aspects of gravitation. Foremost of these is the prediction that gravitation propagates through space-time in the form of quadrupolar gravitational waves, which travel at the same speed as the speed of light. There has been indirect evidence for the existence of gravitational waves through the observation of the decay of the orbit of co-orbiting binary pulsars [1]. The rate of loss of energy in the system matches exactly with the prediction of how much energy is taken away through quadrupolar gravitational radiation. However, these waves have not yet been directly detected. There is a prodigious effort underway to construct enormous, very expensive gravitational wave detectors, which would be able to directly detect gravitational waves through observing their effects on the contraction and expansion of the lengths of the arms of massive interferometers [2].

In this article, we propose an experiment, which as a consequence of the expected finite speed of propagation of changes in the gravitational field, allows us to measure this speed. We suggest that the speed of gravitational phenomena can be measured through table top experiments. Our analysis depends on the idea that if one is able to detect the static gravitational field of a massive body in the laboratory, then moving that body induces detectable changes in the gravitational field that propagate to the detector with a finite delay.

Suppose we have a detector, which reacts to the gravitational field. If we change the gravitational field by moving masses in the neighbourhood of the detector, the detector will register the change. However, the change detected at the detector occurs after a certain delay, that equals to the travel time for the gravitational change to propagate from the mass to the detector. This delay is equal to the distance between the body and the detector divided by

the speed of propagation. Moving just one body in the presence of a detector simply introduces the corresponding changes in the gravitational field at the position of the detector at the appropriately advanced time. However, with two or more bodies moving in the presence of the detector, it is easy to conceive of a situation where the changes in the gravitational field at the position of the detector will contribute constructively or destructively. The interference will depend on the relative distance of the two bodies from the detector. Observing this interference should be possible and would allow for the determination of the speed of propagation of the changes in the gravitational field based on the model of general relativity, as we will explain in the rest of this article. Throughout this article we will use the symbol v_g to denote the speed of gravity, expecting full well that $v_g = c$, the speed of light.

2 Theoretical analysis

Consider the following geometry. A stationary detector of mass m at position \vec{x} is separated from a moving point source of mass M at $\vec{r}(t)$, hence they are separated by a distance $r = |\vec{x} - \vec{r}(t)|$. The gravitational force on the detector is given by Newton's law as

$$|\vec{F}| = \frac{GmM}{|\vec{x} - \vec{r}(t)|^2}. \quad (1)$$

However this expression is not actually correct. In fact, the force at position \vec{x} and at time t corresponds to the Newton expression but with the source mass at the retarded position $\vec{r}(t')$, where $t' = t - |\vec{x} - \vec{r}(t')|/v_g$. We will give a proof of this in the next section, but first we will extract some useful consequences assuming this delay.

If just one mass moves in the presence of the detector, the effects are felt at the detector with an overall delay, and it is difficult to conceive of how to measure this delay. If we arrange two masses, M_i , placed along the x axis, at distances $r_i(t)$, for $i = 1, 2$, on either sides of the detector, which is placed at the origin, then the x component of the force on the detector is given by

$$F_x(x, t) = \frac{GmM_1}{(r_1(t'_1))^2} - \frac{GmM_2}{(r_2(t'_2))^2} \quad (2)$$

where $t'_i = t - r_i(t'_i)/v_g$. Taking $r_i(t) = R_i + \delta_i \sin \omega t \equiv R_i(1 + \epsilon_i \sin \omega t)$ and assuming $|\epsilon_i| \ll 1$, then $t'_i = t - R_i(1 + \epsilon_i \sin \omega t'_i)/v_g \approx t - R_i/v_g + o(\epsilon_i)$. This

yields

$$\begin{aligned}
F_x(x, t) &= \frac{GmM_1}{(R_1(1 + \epsilon_1 \sin \omega(t'_1)))^2} - \frac{GmM_2}{(R_2(1 + \epsilon_2 \sin \omega(t - R_2/v_g)))^2} \\
&\approx \frac{GmM_1}{R_1^2} - \frac{GmM_2}{R_2^2} + \\
&+ \frac{GmM_2}{R_2^2} 2\epsilon_2 \sin \omega(t - R_2/v_g) - \frac{GmM_1}{R_1^2} 2\epsilon_1 \sin \omega(t - R_1/v_g) \quad (3)
\end{aligned}$$

to first order in ϵ_i . Let us first analyze the most evident case, $\frac{GmM_1}{R_1^2} = \frac{GmM_2}{R_2^2} \equiv \frac{GmM}{R^2}$ and $\epsilon_1 = \epsilon_2 \equiv \epsilon$. Then we find

$$F_x(x, t) = \frac{GmM}{R^2} 2\epsilon (\sin \omega(t - R_2/v_g) - \sin \omega(t - R_1/v_g)) \quad (4)$$

to first order in ϵ . Using the simple trigonometric identity, $\sin a - \sin b = 2 \sin \frac{a-b}{2} \cos \frac{a+b}{2}$, gives

$$F_x(x, t) = \frac{GmM}{R^2} 4\epsilon \sin(\omega(R_1 - R_2)/v_g) \cos(\omega(t - (R_1 + R_2)/v_g)). \quad (5)$$

This expression is worth emphasizing. The relative delay, $\Delta t = (R_1 - R_2)/v_g$ tempers the forced oscillations induced by the cosine term. There is an overall delay, $(R_1 + R_2)/v_g$, but it is difficult to see or squarely unobservable. Putting some physical numbers into the formula, for masses of the order of kilograms and distances of the order of meters, the factor in front, $\frac{GmM}{R^2} \approx 10^{-11}$ Newtons. Although very small, it was perfectly measurable even in the time of Cavendish. We now have a further attenuation of the amplitude of the force by $\epsilon \sin(\omega(R_1 - R_2)/v_g)$. The force is periodically varying with temporal dependence $\cos(\omega(t - (R_1 + R_2)/v_g))$. If we imagine $\epsilon \approx 1/10$, $\omega \approx 10^4$ and $\Delta t \approx 10^{-9}$, we find $\epsilon \sin(\omega(R_1 - R_2)/v_g) \approx 10^{-6}$. Thus, the effect will be observable if the detector corresponds to an oscillator of resonant frequency ω , and of sufficiently high Q value. Resonant mechanical detectors, such as torsion balances and micro-cantilevers are known to have Q values of the order of 10^5 [3, ?]. Therefore, with specifically designed experiments, it does not seem to be out of the question that the effect could be observable.

Taking the general case, $\frac{GmM_i}{R_i^2} = F \pm \delta F/2$ and $\epsilon_i = \epsilon \pm \delta\epsilon/2$, with the \pm signs correlated with $i = 1$ or 2 , and allowing for a possible small, intrinsic

asynchronicity in the motion of the two masses δt , we get (advancing the time argument and expanding to first order in $\omega(\Delta t - \delta t)$)

$$\begin{aligned}
F_x(x, t + (R_1 + R_2)/v_g) &= \\
&= \delta F + (F\epsilon + \delta F\delta\epsilon/4)2\sin\omega((R_1 - R_2)/v_g - \delta t)\cos(\omega t) \\
&\quad - (\delta F\epsilon + F\delta\epsilon)\cos\omega((R_1 - R_2)/v_g - \delta t)\sin(\omega t) \\
&\approx \delta F + (F\epsilon + \delta F\delta\epsilon/4)2\omega(\Delta t - \delta t)\cos(\omega t) - (\delta F\epsilon + F\delta\epsilon)\sin(\omega t) \\
&= \delta F + (A^2 + B^2)^{1/2}\cos(\omega t + \alpha). \tag{6}
\end{aligned}$$

Here $A = (F\epsilon + \delta F\delta\epsilon/4)2\omega(\Delta t - \delta t)$ and $B = (\delta F\epsilon + F\delta\epsilon)$ and $\tan\alpha = B/A$.

This expression is a function of three parameters, δF , $\delta\epsilon$ and δt . These parameters should be thought of as adjustable experimental parameters that can be brought to zero by making changes in the experimental apparatus. To make $\delta F = 0$ and $\delta\epsilon = 0$, we simply have to adjust two parameters, the equilibrium position and the amplitude of the oscillations of one of the masses, until the amplitude of the driving force, and consequently that of the resonating detector, becomes minimal. Experimentally, this requires a search in a two dimensional parameter space. When attained, we have $B = 0$, $\alpha = 0$ and $A = F\epsilon 2\omega(\Delta t - \delta t)$. It is further possible to adjust the intrinsic asynchronicity so that $\delta t = \Delta t$ which makes the entire driving force vanish. But this procedure then just gives us a measure of Δt . We can then directly extract the speed of gravity by measuring R_1 and R_2 . Indeed, $v_g = (R_1 - R_2)/\Delta t$.

3 Mathematical justification

Assuming the expected relativistic invariance of the propagation of gravitational effects, the field at the detector can be calculated from the gravitational analog of the Lienard-Wiechert [6] potentials for point sources. Indeed, the empty space, matter free solution to the Einstein equations for the gravitational metric field is given by the Minkowski metric

$$[g_{\mu\nu}] = [\eta_{\mu\nu}] = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}. \tag{7}$$

and adding a small, perturbing source yields

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}. \tag{8}$$

We get in first approximation, in the harmonic gauge [7]

$$\square h_{\mu\nu} = \frac{16\pi G}{v_g^2} S_{\mu\nu} \quad (9)$$

and $S_{\mu\nu} = T_{\mu\nu} - (1/2)T^\lambda_\lambda \eta_{\mu\nu}$, and $\square = \partial^2/\partial t^2 - v_g^2 \vec{\nabla} \cdot \vec{\nabla}$ with all derivatives taken with respect to the flat coordinates. v_g is the speed of gravity, which is of course expected to be the same as the speed of light.

The solution to this equation is obtained by using the retarded Green's function

$$h_{\mu\nu}(\vec{x}, t) = \frac{-4G}{v_g^2} \int d^3x' dt' S_{\mu\nu}(\vec{x}', t') \frac{\delta(t - t' - |\vec{x} - \vec{x}'|/v_g)}{|\vec{x} - \vec{x}'|} \quad (10)$$

where the delta function imposes the retarded time condition, *viz.* the change in the metric, $h_{\mu\nu}$, at position \vec{x} at time t is given by Newton's law for the point mass at position \vec{x}' and at time t' where $t' = t - |\vec{x} - \vec{x}'|/v_g$. Clearly $|\vec{x} - \vec{x}'|/v_g$ represents the time it takes for the signal to travel from \vec{x}' to \vec{x} .

However, we will be considering a mass of finite size, moving rigidly. Suppose a point in the mass moves according to the trajectory $\vec{r}(t)$, then all other points in the body are displaced with respect to this point by a time independent displacement, \vec{r} . The energy momentum tensor for a point mass \tilde{m} is given by

$$T_{\mu\nu}(\vec{x}, t) = \frac{p_\mu(t)p_\nu(t)}{E(t)} \delta^3(\vec{x} - \vec{r}(t)) \approx \tilde{m} \beta_\mu(t) \beta_\nu(t) \delta^3(\vec{x} - \vec{r}(t)) \quad (11)$$

where the last approximation is valid in the non relativistic limit, $\beta_\mu \equiv (1, v_i/v_g)$. Then $S_{\mu\nu} = \tilde{m}(\beta_\mu(t)\beta_\nu(t) - (1/2)\eta_{\mu\nu})\delta^3(\vec{x} - \vec{r}(t))$, replacing $\tilde{m} \rightarrow d^3r\rho$, $\vec{r}(t) \rightarrow \vec{r}(t) + \vec{r}$ and integrating over d^3r , we get

$$h_{\mu\nu}(\vec{x}, t) = \frac{-4G}{v_g^2} \int d^3r \rho(\vec{r}) \int dt' (\beta_\mu(t')\beta_\nu(t') - (1/2)\eta_{\mu\nu}) \times \quad (12)$$

$$\times \int d^3x' \frac{\delta^3(\vec{x}' - \vec{r}(t') - \vec{r}) \delta(t - t' - |\vec{x} - \vec{x}'|/v_g)}{|\vec{x} - \vec{x}'|} \quad (13)$$

$$= \frac{-4G}{v_g^2} \int d^3r \rho(\vec{r}) \int dt' (\beta_\mu(t')\beta_\nu(t') - (1/2)\eta_{\mu\nu}) \times \quad (14)$$

$$\times \frac{\delta(t - t' - |\vec{x} - \vec{r}(t') - \vec{r}|/v_g)}{|\vec{x} - \vec{r}(t') - \vec{r}|}. \quad (15)$$

Now the assumption that we must make is clear, if $|\vec{r}| \ll |\vec{x} - \vec{r}(t')|$ then we can neglect \vec{r} to lowest approximation. Writing $M = \int d^3r \rho(\vec{r})$, we find

$$h_{\mu\nu}(\vec{x}, t) = \frac{-4GM}{v_g^2} \int dt' m(\beta_\mu(t')\beta_\nu(t') - (1/2)\eta_{\mu\nu}) \frac{\delta(t - t' - |\vec{x} - \vec{r}(t')|/v_g)}{|\vec{x} - \vec{r}(t')|}. \quad (16)$$

Then for example, we find, dropping all velocity dependent terms,

$$h_{00}(\vec{x}, t) = \frac{-2GM}{v_g^2 |\vec{x} - \vec{r}(t')|} = h_{11}(\vec{x}, t) = h_{22}(\vec{x}, t) = h_{33}(\vec{x}, t) \quad (17)$$

where t' is the retarded time satisfying $t' = t - |\vec{x} - \vec{r}(t')|/v_g$, which is the lowest order approximation to the Schwarzschild-like metric

$$v_g^2 d\tau^2 = \left(1 - \frac{2GM}{v_g^2 |\vec{x} - \vec{r}(t')|}\right) v_g^2 dt^2 - \frac{1}{\left(1 - \frac{2GM}{v_g^2 |\vec{x} - \vec{r}(t')|}\right)} (dr^2 + r^2 d\Omega^2). \quad (18)$$

This means that the correct potential is simply the usual Newtonian potential but corresponding to the source at the retarded position, as we have used above.

4 Perspectives for measurements

Here we analyze the perspectives for the possibilities of observing the effect and determining the speed of gravity.

4.1 Torsion balances, cantilevers and other mechanical detectors

Torsion balances are the oldest, most venerable detectors of gravitational phenomena [3]. The greatest problem, however, is that the amplitude of the swing of the torsion balance can be appreciable, in fact non-negligible compared to the amplitude of the oscillating masses. In this case, one must take into account the movement of the detector, which we have assumed to be negligible. Micro-cantilevers [4],[5] on the other hand, have very little intrinsic movement. Superconducting gravimeters [8] may well be better suited. Such devices levitate a superconducting test mass in the magnetic field of persistence currents of superconducting loops. As the currents are

exactly stable, because superconducting persistence currents do not decay, the levitating force has no variation, and hence the system is ideally suited to detecting changes in the gravitational field. Such devices could be ideally suited for detecting our proposed effect.

In the end, the Q value of the resonant detector will govern whether or not it will be able to detect the effect. Q values of mechanical detectors lie in the range 10^4 to 10^5 which is at the limit of what is required. We have a driving force with an amplitude that is approximately 10^{-6} smaller than what is an easily detectable gravitational force. However, we imagine our masses made of the most dense materials available, a kilogram of Osmium corresponds to a sphere of radius 3.25 centimetres. Gold is not that far off. Then the distance to the detector could be cut to about 30 centimetres while the oscillation amplitude could be taken to be about 3cm. Then the force is increased by a factor of 10. Thus, specialized detectors and experimental set up seem to indicate that detecting our effect is not out of the realm of possibility.

4.2 Gravitational wave interferometers

The very large arm, interferometric gravitational wave detectors [2] could be conceivably used to detect our predicted time delay. Indeed, at first sight, they seem ideally suited. The experimental idea would be to bring two masses in the neighbourhood of one of the arms and oscillate them accordingly to obtain a time dependent delay in the light travel time in that arm due to the Shapiro time delay [9]. However, there are two problems to this implementation, one of which is surmountable, while the other does not seem to be.

4.2.1 Null condition

The first problem has to do with the fact that the effect that we wish to detect is encoded in the metric. The Shapiro time delay occurs because the null geodesics for light, in the presence of a mass are lengthened due to the bending of light. But the null geodesics are properties of the metric of the theory. The metric of the theory contains the Newtonian gravitational potential, and not the gravitational force. Bringing two masses on either side of the trajectory of a light ray, the gravitational potentials at the position of

the light ray simply add. The gravitational potential is given by

$$\Phi = -\frac{GM}{r} \quad (19)$$

which is negative definite. Therefore, the null condition is not easily obtained for the potential. In contradistinction, if the detector is sensitive to force the null condition is easy to establish, the forces cancel being in opposite directions. It would also be possible for the potential, if negative mass existed, the potential of a negative mass has the opposite sign from that of a positive mass.

However, it is easy to conceive of an effective negative mass. The potential of a spherical body of uniform density, with a smaller spherical cavity in it, is exactly as if the volume of the cavity was simultaneously filled with equal, positive and negative mass density. Thus the mass density effectively corresponds to a full, solid sphere of positive mass density, and a smaller sphere located at the position of the cavity filled with negative mass density. If we write M_0 as the positive mass of the sphere without the cavity, and $-M_1$ the mass of the cavity as if it were filled with negative mass, and \vec{r}_0 the position of the centre of the cavity, the potential at a position \vec{x} is given by

$$\Phi(\vec{x}) = -\frac{GM_0}{|\vec{x} - \vec{r}|} + \frac{GM_1}{|\vec{x} - (\vec{r} + \vec{r}_0)|} \quad (20)$$

This is simply the linear superposition of the potential of the full, cavity free, larger spherical body of positive mass, plus the potential of the smaller spherical cavity but now filled with negative mass.

Now imagine rotating the spherical body on an axis that passes through its centre, which is a distance R_0 from the detector, but an axis that is perpendicular to the separation direction to the detector. Such a source would produce a time dependent negative mass potential at the position of the detector. The total potential at the position of the detector, adding in the potential of an ordinary sphere of mass M_2 which is moving in a mirror reflection of the motion of the cavity, using the conventions from Equation

(3), is given by

$$\begin{aligned}
\Phi &= -\frac{GM_0}{R_0} + \frac{GM_1}{R_1((1 + \epsilon_1 \sin(\omega t'_1))^2 + \epsilon_1^2 \cos^2(\omega t'_1))^{(1/2)}} \\
&\quad - \frac{GM_2}{R_2((1 + \epsilon_2 \sin(\omega t'_2))^2 + \epsilon_2^2 \cos^2(\omega t'_2))^{(1/2)}} \\
&= -\frac{GM_0}{R_0} + \frac{GM_1}{R_1(1 + 2\epsilon_1 \sin(\omega t'_1) + \epsilon_1^2)^{(1/2)}} \\
&\quad - \frac{GM_2}{R_2(1 + 2\epsilon_2 \sin(\omega t'_2) + \epsilon_2^2)^{(1/2)}}. \tag{21}
\end{aligned}$$

Interestingly, the fluctuations in the transverse direction only contribute at second order, and in fact in a time independent manner. Expanding to first order we find

$$\begin{aligned}
\Phi &\approx -\frac{GM_0}{R_0} + \frac{GM_1}{R_1}(1 + \epsilon_1 \sin(\omega t'_1) + o(\epsilon_1^2)) \\
&\quad - \frac{GM_2}{R_2}(1 + \epsilon_2 \sin(\omega t'_2) + o(\epsilon_2^2)) \tag{22}
\end{aligned}$$

Now taking $M_1/R_1 = M_2/R_2 = M/R$, $\epsilon_1 = \epsilon_2 = \epsilon$, creates exactly the null condition for the time dependent part of the gravitational potential at the position of the detector, and thus solving this problem¹

$$\Phi \approx -\frac{GM_0}{R_0} + \frac{GM\epsilon}{R}(\sin(\omega t'_1) - \sin(\omega t'_2)). \tag{23}$$

Also, it should be pointed out that this expression is strictly the Newtonian potential. We do realize that rotating extended bodies does not correspond to rigid body translations. For example, the fact that the larger spherical body is rotating does affect the energy momentum tensor of the matter fields and the consequent metric. Additionally, rotating the cavity and the mirror reflected mass do not correspond to rigid translations of these masses. The full metric tensor in the Einsteinian theory contains additional terms which are due to the rotating, though stationary, energy-momentum distributions.

¹We thank G. Kunstatter for pointing out that it is also possible to obtain the null condition by taking ordinary masses but simply changing the phase relation of the oscillation by π . Specifically if M_1 was just an ordinary mass (not a cavity) this changes the sign of the second term on the RHS of Equation (22). Then we must redefine M_0 and R_0 , and if we take $\epsilon_1 = -\epsilon_2 = \epsilon$, we achieve the null condition.

The metric will have Kerr [10] geometric type corrections, and the dynamics as given by the Newtonian potential, are only an approximation. These corrections are, in any case, very small as the velocities involved are non-relativistic.

4.2.2 Metric, the dimensionless potential and the calculation of the Shapiro time delay

However, the second problem is that the effect is unfortunately undetectably small, and this problem seems insurmountable. It is not the simple Newtonian gravitational potential Φ that appears in the metric, but Φ/v_g^2 . The potential for kilogram sized masses and meter sized distance is of the order of 10^{-11} Joules/kilogram = 10^{-11} m²/s², which is perhaps measurable, but what appears in the metric is the dimensionless potential obtained by dividing by $v_g^2 \sim 10^{17}$ m²/s² which implies the potential in the metric is in fact, of the order of $\sim 10^{-28}$. Thus, the Shapiro time delay corresponds to wavelength dephasing by $MG/v_g^2 \sim 10^{-28}m$ which is not measurable. Specifically, the metric at \vec{x}, t for a mass M at position $\vec{r}(t')$ (t' the retarded time), in our approximation and in cartesian coordinates, is given by

$$v_g^2 d\tau^2 = \left(1 - \frac{2GM}{v_g^2 |\vec{x} - \vec{r}(t')|}\right) v_g^2 dt^2 - \left(1 + \frac{2GM}{v_g^2 |\vec{x} - \vec{r}(t')|}\right) (dx^2 + dy^2 + dz^2). \quad (24)$$

There will be an additional contribution from the spherical cavity with effectively negative mass, which will simply subtract from that of the positive mass, and hence we shall not treat explicitly.

Consider the arm of the interferometer along the x axis and the separation of the mass from the interferometer along the y axis. Nothing depends on z , and it can be dropped. Thus $\vec{x} = x\hat{x}$ and $\vec{r}(t') = r(t')\hat{y}$ and $|\vec{x} - \vec{r}(t')| = (x^2 + r(t')^2)^{1/2}$. The light travels on null geodesics, $d\tau^2 = 0$, which gives

$$\left(1 - \frac{2GM}{v_g^2 (x^2 + r(t')^2)^{1/2}}\right) v_g^2 dt^2 = \left(1 + \frac{2GM}{v_g^2 (x^2 + r(t')^2)^{1/2}}\right) dx^2 \quad (25)$$

that is (in lowest approximation)

$$v_g dt = \left(1 + \frac{GM}{v_g^2 (x^2 + r(t')^2)^{1/2}}\right) dx. \quad (26)$$

The first term gives the usual Euclidean result while the second term gives the additional time that it takes - the Shapiro time delay - for the light to travel over the distance dx at the space-time position \vec{x}, t . We should not forget that $t = t(x)$, (or vice versa) implicitly a function of x defined by Equation (26).

In the experimental setting, if the oscillating masses are situated at L_1 and L_2 from the semi-reflecting mirror and the end mirror, respectively, of one arm of the interferometer, we must integrate over x from 0 to L_1 , add to it the integration from 0 to L_2 and then multiply by 2, to get the total light travel time. We assume $r(t') = r_0(1 + \epsilon \sin \omega t')$, and we take $\epsilon \rightarrow 0$. Then we get $t' \approx t - (x^2 + r_0^2)^{1/2}/v_g + o(\epsilon)$, and we have the time delay, on top of the Euclidean value $v_g T_L = L$, (in obvious notation and to be taken first order in ϵ)

$$\begin{aligned} v_g \Delta T_L &= \int_0^L \frac{GM}{v_g^2 (x^2 + r_0^2 (1 + \epsilon \sin \omega (t - (x^2 + r_0^2)^{1/2}/v_g))^2)^{1/2}} dx \\ &\approx \int_0^L \frac{GM}{v_g^2} \left(\frac{1}{(x^2 + r_0^2)^{1/2}} - \frac{(r_0^2 \epsilon \sin \omega (t - (x^2 + r_0^2)^{1/2}/v_g))}{(x^2 + r_0^2)^{3/2}} \right) dx \end{aligned} \quad (27)$$

The t appearing in Equation (27) is a function of x , but since the delay is already first order, we can use the zero order expression $t(x) = x/v_g$. There is no point in doing this further calculation. To get an estimate of the order of the size of the RHS in Equation (27) we replace the $\sin \approx 1$, assume $r_0 \ll L$ and use simple dimensional analysis replacing the integration by a simple factor L to get

$$v_g \Delta T_L \approx \frac{GM}{v_g^2} \left(1 - \frac{r_0^2 \epsilon}{L^2} \right) \quad (28)$$

The length L is almost 11km for the case of Ligo [2], thus the second term (and in actual fact the time dependent part) is incompletely negligible. Thus we find $v_g \Delta T_L \approx MG/v_g^2 \sim 10^{-28}m$ which is simply too small to be measurable.

Nevertheless, it may still be practical to use these interferometers by making the mirrors oscillate due to the moving external masses. Indeed, the end mirror is so highly isolated from any mechanical vibrations, that even quantum mechanical uncertainty in its position has to be taken into account, [11]. Thus oscillations induced in the end mirror directly by gravitational force might be easily measurable.

4.3 Quantum resonance detection, quantum interferometric devices

Such devices are sensitive to the gravitational potential. We mention that they have exceptionally large Q values, of the order of 10^{10} [12]. We will mention only one such system, that corresponding to the experiment q-Bounce [13], [14]. Here ultra-cold neutrons are trapped in a container. They are cold enough so that they do not penetrate the walls of the container. They only feel the gravitational potential of the earth. The quantum mechanical wave-functions in a linear gravitational potential are the Airy functions, and implementing the boundary conditions gives discrete energy levels that are separated by pico-eV. The experiment has already succeeded in observing resonant transitions between the energy levels. Pico-eV corresponds to frequencies of a few hundreds of Hertz. This is ideally suited for physically oscillating macroscopic masses. Indeed, if the potential was made to oscillate at the resonant frequency of the transition, it could be possible to induce transitions between the neutron energy levels. The gravitational potential of the earth is approximately 10 m/sec^2 . The gravitational potential of kilogram sized masses at distances of the order of meters is approximately 10^{-11} m/sec^2 . Thus the Q value of the system must be in the same range, $10^{11} \sim 10^{12}$ in order for the effect to be observable.

5 Conclusion

We have proposed a simple experiment, which would in principle allow the determination of the speed of propagation of gravity. It would be an important, confirmation and validation of Einstein's general relativity if one could observe that gravitational disturbances are not instantaneously transmitted throughout all of space but propagate through it at a finite velocity, and indeed, to measure that velocity.

Acknowledgements

We thank Hartmut Abele, Luca Fabbri, Gabor Kunstatter, Richard MacKenzie, Suneeti Phadke and Viktor Zacek for useful discussions and NSERC of Canada for partial financial support.

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